Pursuit Problems

Imperial College London Mathematics School

Mohamed, Jacob, Bright, Domenico

Imperial College London

Mathematics School

Abstract

Our paper discusses the different possibilities of pursuit problems, and the various outcomes of specific examples. We coded multiple situations for the “mice problem”, and made observations on how the resultant spiral changed for different numbers and spacing of mice. We looked at how factors such as the time taken, and the area of the spiral varied. We then compared this to the phenomena of ant mills forming in nature, and answered the question of whether the formation of these “death spirals” was inevitable in the mice problem. We also investigated how rules of pursuit could be observed in groups, through both “boids” and crowd behaviour, whilst looking at their various real-world applications

1. Radiodrome

A radiodrome is a simple pursuit curve, created by one object (the predator) chasing another (the prey). The prey moves in a straight line and the predator always moves towards the prey. If they are moving at the same speed this leads to the predator never catching the prey and instead following it indefinitely. However, the predator can, in certain situations, catch the prey if it anticipates its movement. The radiodrome problem was first investigated by Pierre Bouguer in 1732 in an article on navigation.[Bou32]

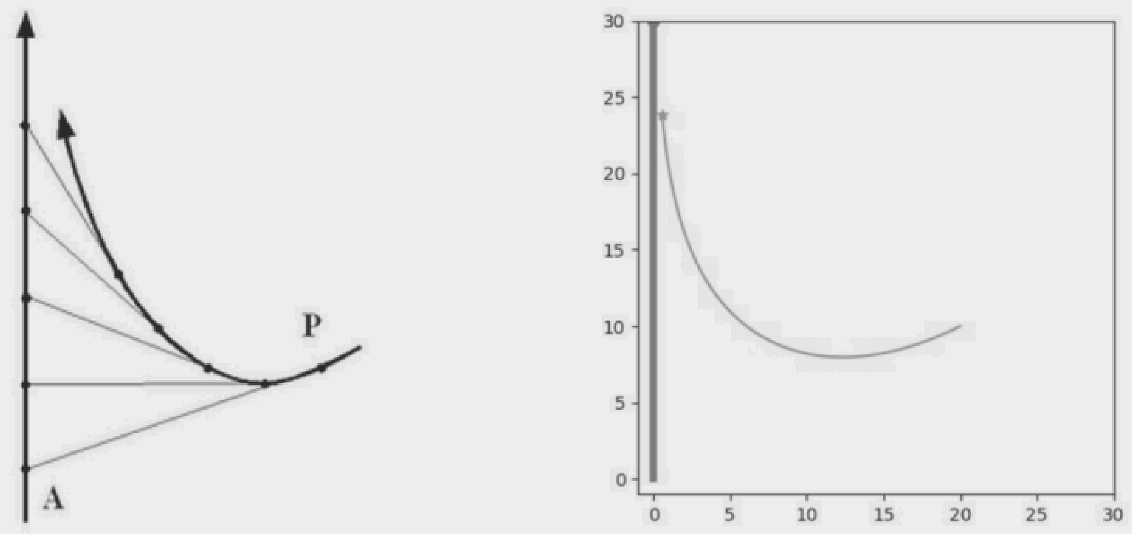

Figure 1: Example of a pure pursuit radiodrome that we found online compared to one we simulated ourselves [WB05].

2. Ant Mill

The ant mill is a type of pursuit problem and self organisation which occurs in nature when an ant loses the pheromone track from the ant in front of it. Once an ant loses track of the pheromone trail, it will follow the ant in front. This results in all the ants behind it following the ant in front, leading to the formation of a spiral known as an ‘ant mill’. This phenomenon is often also dubbed a ‘death spiral’, because the ants circle around an area, converging over time on a central point - before they eventually die from exhaustion. The ant mill was first described by William Beebe in 1921, who observed an ant mill 370m in circumference. [Bee21]

Figure 2: Ant Mill [Cle21]

3. Mice Problem

The mice problem is a well known mathematical dilemma, where a number of mice, or any entities, start at the corners of a polygon, and move towards their neighbour. This can cause different effect depending on the number of sides, length of sides and shape of the polygon. By using Python, we managed to simulate the different potential outcomes for this problem. We began by initially simulating the possibilities with different sided regular polygons, which in turn affect the number of mice involved. In these simple examples with regular polygons, the mice form the ‘death spiral’ and converge on the centre of mass of the shape (the centroid).

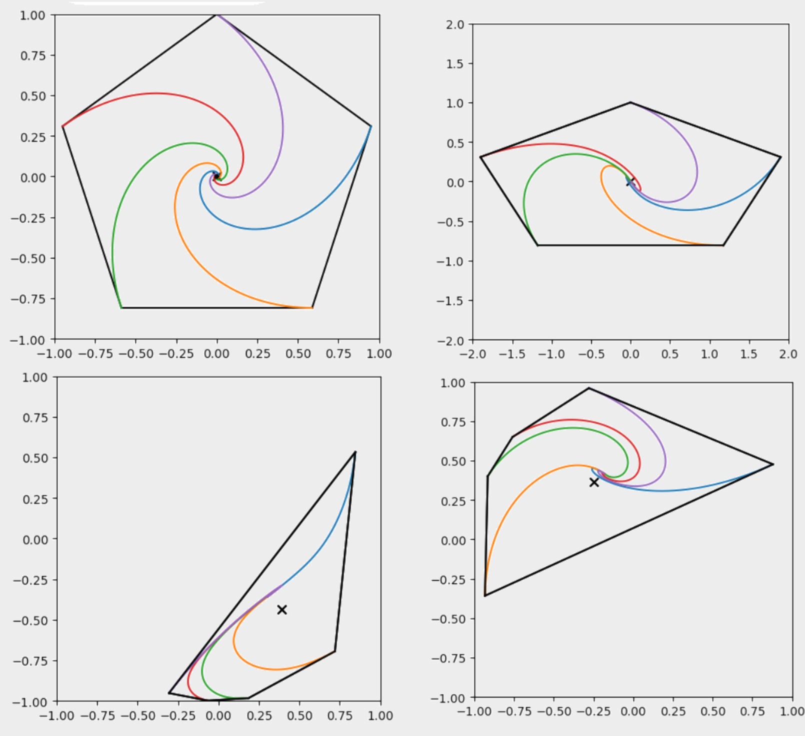

Figure 3: The mice problem with the mice starting at the corners of regular, stretched and randomly generated pentagons.

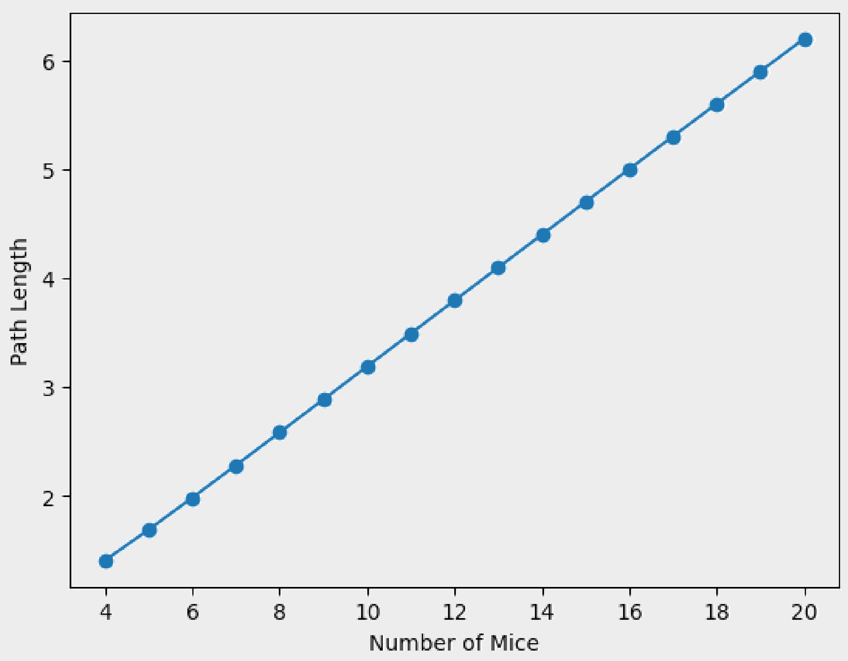

Figure 4: Path length of any one mouse for regular polygons from 4 to 20 sides

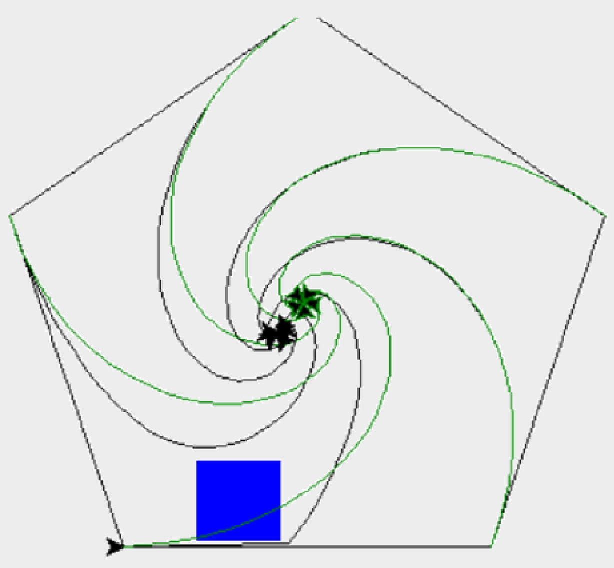

Figure 5: Mice problem with an obstacle in the path of one of the mice.

4. Boids

Boids is a program which was created in order to simulate birds flocking. It is made using many small entities, birds, which all move based on 3 forces, a force of cohesion (which brings them closer together), a force of alignment (which causes them to move in a similar direction), and a force of separation (which pushes them away from each other). We observed the boids movements and saw that they mostly travelled in elongated hexagons, with a ratio of long to short sides 2:1. We deduced that the reason behind this is most likely due to the shape of the boids, being isosceles triangles with the ratio of long to short sides, 2:1.

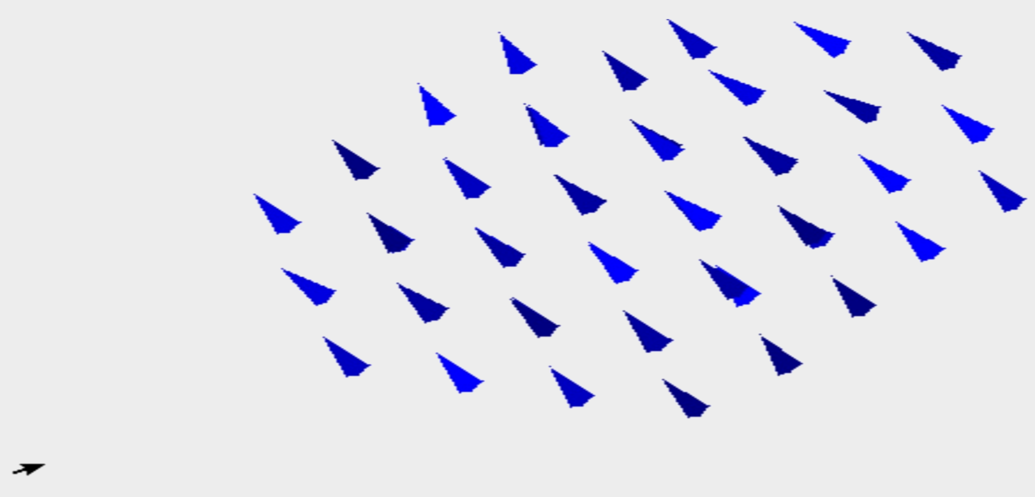

Figure 6: Our attempt at simulating boids.

Conclusions

From our research and coded results we found various ways of simulating crowd dynamics, both in nature and artificially. In nature, this is seen with the formation of ant mills, or ‘death spirals’. We simulated these through artificial means in our testing under different restraints of the mice problem. We also modelled crowd dynamics in our boids simulations. In conclusion, this project showed us that simple rules for individuals can definitely generate complex group behaviour, and how this may be applied to real life scenarios.

References

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