

## Imperial College London Mathematics School Admissions Test

Content statements are copied from GCSE Subject Level Conditions and Requirements for Mathematics<sup>1</sup>. Numbering is as in the Ofqual document; gaps in numbering indicate content has been omitted below.

### Number

#### **Structure and calculation**

1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals)
4. use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem
5. apply systematic listing strategies
6. use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5;
7. calculate with roots, and with integer indices
8. calculate exactly with fractions, and multiples of  $\pi$ ;
9. calculate with and interpret standard form  $A \times 10^n$ , where  $1 \leq A < 10$  and  $n$  is an integer.

#### **Fractions, decimals and percentages**

10. work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and  $\frac{7}{2}$  or 0.375 or  $\frac{3}{8}$ );
11. identify and work with fractions in ratio problems
12. interpret fractions and percentages as operators.

**Measures and accuracy**

13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding
16. apply and interpret limits of accuracy,

**Algebra**

**Notation, vocabulary and manipulation**

1. use and interpret algebraic notation, including:
  - $ab$  in place of  $a \times b$
  - $3y$  in place of  $y + y + y$  and  $3 \times y$
  - $a^2$  in place of  $a \times a$ ,  $a^3$  in place of  $a \times a \times a$ ,  $a^2b$  in place of  $a \times a \times b$
  - $\frac{a}{b}$  in place of  $a \div b$
  - coefficients written as fractions rather than as decimals
  - brackets
2. substitute numerical values into formulae and expressions, including scientific formulae
3. understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors
4. simplify and manipulate algebraic expressions (including those involving surds **and algebraic fractions**) by:
  - collecting like terms
  - multiplying a single term over a bracket
  - taking out common factors
  - expanding products of two binomials
  - factorising quadratic expressions of the form  $x^2 + bx + c$ , including the difference of two squares; **factorising quadratic expressions of the form  $ax^2 + bx + c$**
  - simplifying expressions involving sums, products and powers, including the laws of indices

5. understand and use standard mathematical formulae; rearrange formulae to change the subject
6. know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments
7. where appropriate, interpret simple expressions as functions with inputs and outputs;

### **Graphs**

8. work with coordinates in all four quadrants
9. plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form  $y = mx + c$  to identify parallel lines; find the equation of the line through two given points, or through one point with a given gradient
10. identify and interpret gradients and intercepts of linear functions graphically and algebraically
11. identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically
12. recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function  $y = \frac{1}{x}$  with  $x \neq 0$ ,
14. plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

### **Solving equations and inequalities**

17. solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
18. solve quadratic equations (**including those that require rearrangement**) algebraically by factorising, **by completing the square**; find approximate solutions using a graph
19. solve two simultaneous equations in two variables (linear/linear) algebraically; find approximate solutions using a graph
21. translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.
22. solve linear inequalities in one variable; represent the solution set on a number line,

### **Sequences**

23. generate terms of a sequence from either a term-to-term or a position-to-term rule
24. recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions ( $r^n$  where  $n$  is an integer, and  $r$  is a rational number  $> 0$ )
25. deduce expressions to calculate the  $n$ th term of linear sequences.

### **Ratio, proportion and rates of change**

1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
2. use scale factors, scale diagrams and maps
3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
4. use ratio notation, including reduction to simplest form
5. divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
6. express a multiplicative relationship between two quantities as a ratio or a fraction
7. understand and use proportion as equality of ratios
8. relate ratios to fractions and to linear functions
9. define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
10. solve problems involving direct and inverse proportion, including graphical and algebraic representations
11. use compound units such as speed, rates of pay, unit pricing, density and pressure
12. compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors
13. understand that  $X$  is inversely proportional to  $Y$  is equivalent to  $X$  is proportional to  $\frac{1}{Y}$ ;  
interpret equations that describe direct and inverse proportion
14. interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion

## Geometry and measures

### ***Properties and constructions***

1. use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description
3. apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
4. derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
5. use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
6. apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
7. identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional scale factors)
9. identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
11. solve geometrical problems on coordinate axes
12. identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
13. construct and interpret plans and elevations of 3D shapes.

### ***Mensuration and calculation***

14. use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
15. measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
16. know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
17. know the formulae: circumference of a circle =  $2\pi r = \pi d$ , area of a circle =  $\pi r^2$ ; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids

18. calculate arc lengths, angles and areas of sectors of circles

19. apply the concepts of congruence and similarity, including the relationships between lengths, in similar figures

20. know the formulae for: Pythagoras' theorem,  $a^2 + b^2 = c^2$ , and the trigonometric ratios,

$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ ,  $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$  and  $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ ; apply them to find angles and

lengths in right-angled triangles in two dimensional figures

21. know the exact values of  $\sin \theta$  and  $\cos \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$  and  $90^\circ$ ; know the exact value of  $\tan \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ$  and  $60^\circ$

### **Vectors**

24. describe translations as 2D vectors

### **Probability**

1. record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees

2. apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments

3. relate relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale

4. apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one

5. understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size

6. enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams

7. construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities

8. calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

### **Statistics**

1. infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling

2. interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped

discrete numerical data, tables and line graphs for time series data and know their appropriate use

4. interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:

- appropriate graphical representation involving discrete, continuous and grouped data
- appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)

5. apply statistics to describe a population

6. use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

Content **in bold type** comes from the higher tier GCSE specification.

### **Style and length of test**

- The test is 1.5 hours long and to be taken without a calculator. Students answer in spaces in the answer booklet.
- The test should assess potential for the maths school.
- The test has 3 sections – each with 20 marks.
  - Section A: 10 multiple choice questions testing understanding of GCSE content; each question is worth 2 marks and has 5 options.
  - Section B: 10 multiple choice questions testing reasoning and thinking using GCSE content; each question is worth 2 marks and has 5 options.
  - Section C: 3 questions with written answers; two at approx. 5 marks each and one at approx. 10 marks, testing presentation of mathematical arguments and problem solving.